The PROMYS Math Circle

Problem of the Week #2

January 27, 2017

1. Find the sum of all four-digit numbers which consist of the digits 1, 2, 3, and 4 in some order, with each digit appearing exactly once. Be sure to explain how you got your answer.

Star of the Week #2
Vicki Do (Everett High School)

Find the sum of all four-digit numbers which consist of the digits 1, 2, 3, *and* 4 *in some order, with each digit appearing exactly once. Be sure to explain how you got your answer.*

Thanks to all of you who submitted solutions to the second Problem of the Week (POW #2). There were lots of good answers. Most people figured out that the sum of all four-digit numbers consisting of the digits 1, 2, 3, 4 is 66660.

Vicki Do of Everett High School has been chosen as this week's "Star of the Week" because she tried different approaches, didn't give up when her first attempt didn't work out nicely, and then found a clever way to see the right answer without actually doing all the computation. If you look at her solution, you can see her thought process. She started by trying the "brute force" method – she just wrote down all the numbers in a column (there were 24 different numbers) and tried to add them up. This was hard work and the answer she got didn't look right somehow, so she kept thinking about it. She looked for a conceptual way of understanding her list of 24 numbers and realized that each column had all four of the digits $1, 2, 3, 4$ in it, each appearing six times. That's six 1's, six 2's, six 3's and six 4's. So she realized that the sum of each column must be

$$6 \cdot 1 + 6 \cdot 2 + 6 \cdot 3 + 6 \cdot 4 = 6 + 12 + 18 + 24 = 60.$$

So the sum of the ones column is 60 and so also is the sum of tens column, and also the hundreds and thousands columns add up to 60. So putting all the numbers together we have 60 thousands, 60 hundreds, 60 tens, and 60 ones. That gives us

$$60000 + 6000 + 600 + 60 = 66660.$$

That's the right answer, but Vicki's solution is more than just the answer, since she also explains how to *understand* the answer. That's good mathematics!

Enjoy the snow this weekend and send us your ideas on POWs #3 and #4.

Glenn Stevens (Boston University)

Vicki Do

1. Find the sum of all four-digit numbers which consist of the digits 1, 2, 3, and 4 in some order, with each digit appearing exactly once. Be sure to explain how you got your answer.

1234 —
1324 —
1342 —
1432 ✓
1423 ✓
1243 ✓
2134 —
2143 ✓
2431 ✓
2413 ✓
2314 ✓
2341 ✓
3241 —
3421 —
3412 —
3214 —
3124 —
3142 —
4321 ✓
4312 ✓
4231 —
4213 —
4123 —
4132

1234
2134
1342
1423
1324
2314
2341
2431
2413

brute force

$\boxed{67,903}$

6×10^4 ten thousands
6×10^3 thousands
6×10^2 hundreds
6×10^1 tens

$60000 + 6000 + 600 + 60 =$

$\boxed{66660}$

Think how many times will
1, 2, 3, 4 in the
thousands...

$6 \cdot 1 = 6$
$6 \cdot 2 = 12$ } 60
$6 \cdot 3 = 18$
$6 \cdot 4 = 24$