PROMYS

Program in Mathematics for Young Scientists

Boston University

June 27 to August 7, 2010

Program Structure

This summer, Boston University will sponsor an intensive six-week Program in Mathematics for Young Scientists (PROMYS). An ambitious group of approximately 60 high school students will gather on our campus to engage in intensive study of a significant piece of mathematics. They will work with members of our faculty and be assisted by experienced undergraduates who are embarking on their own mathematical careers at some of our nation's finest universities.

Each morning, participants will attend a Number Theory class; more experienced students may also attend an advanced seminar. They will then work independently and in small groups on problem sets distributed at the end of each class meeting. The problems will encourage students to design their own numerical experiments and employ their own powers of observation, as they discover mathematical patterns, formulate and test conjectures, and justify their ideas by devising their own mathematical proofs. In addition, each student will be supervised by a counselor in residence who will always be available for helpful discussions.

Students who find their PROMYS experience especially worthwhile may apply to return for further studies in the following summer. In 2010, we plan to offer advanced seminars in Linear Algebra, Modular Forms, and Geometry and Symmetry. Advanced participants also work on independent research projects in mathematics. Research projects of past summers have ranged from combinatorics, hyperbolic geometry, Ramanujan's q-calculus, continued fractions and geodesics in the Poincaré upper half-plane, quaternion algebras and quadratic forms, to name only a few. Counselors and returning students will also organize their own seminars on topics of interest to members of the program.

Program Goals

Mathematics may well be the most widely misunderstood branch of the sciences. Young people contemplating careers in science find it difficult to imagine what a research mathematician really does. One common image features a mathematician programming a computer to do difficult calculations. Another pictures a lone mathematician working in isolation on ideas so abstruse that no normal person could comprehend them. Neither of these images comes close to capturing the spirit of mathematical inquiry. It is certainly true that many modern mathematicians use computers to perform numerical and geometrical experiments. But this experimental phase is only one component of the mathematical experience, and the use of computers in this phase is the exception, not the rule. Nor is it true that mathematicians work in isolation. Indeed, a distinctive feature of mathematics is the open sharing of ideas within a community nurtured by a common language, shared values, and shared goals.

All too often, mathematics is presented to students as a highly polished and well organized collection of definitions, algorithms, and theorems. The long struggle of many individuals that culminated in this "finished product" remains a hidden secret. Students rarely learn of the dynamic nature of mathematics, nor do they see the creative side. They do not come to understand that mathematics is a thriving field of research activity which is progressing faster today than at any other time in its distinguished history.

These misunderstandings may stem from the fact that mathematics deals so heavily in ideas. In mathematics, perhaps more than in any other science, research is an activity of the mind. The primary goal of the mathematician is to $understand$ – to discover the essential ingredients of complex systems in order to render them simple, to find order within apparent chaos, to draw analogies between different structures, and to find connections between seemingly disparate branches of mathematics and science. To make interesting new contributions in the field of mathematics requires a healthy mix of creativity, experience, and hard work.

We aim to engage young people in the struggle to understand an intricate collection of significant mathematical ideas. PROMYS participants come with unbounded energy and are anxious to grapple with challenging ideas. At the beginning of their investigations, they may sometimes feel lost and perplexed. But through carefully designed problem sets, we hope to subtly direct PROMYS students along productive paths towards understanding – to suggest that they experiment with examples and formulate conjectures, to encourage them to ask good questions, and to help them realize that through careful thought they can penetrate formidable obstacles and invent their own answers to difficult questions. The attitudes acquired through this experience will be far more valuable than the particular topics mastered.

Professor Glenn Stevens
Director of PROMYS

PROMYS gratefully acknowledges the financial support of its sponsors:

- Boston University
- Clay Mathematics Institute/PROMYS partnership
- National Science Foundation
- National Security Agency
- Park City Mathematics Institute
- American Mathematical Society

and many other private contributions from alumni and friends of PROMYS. Please visit our website www.promys.org for links to each of these institutions.

Application form for

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Please complete this form and return it along with an official transcript from your school to: PROMYS, Department of Mathematics, Boston University, 111 Cummington Street, Room 142, Boston, Massachusetts 02215. Application deadline is May 30, 2010.

Name: ____

Address: ____

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Telephone: ____________________ Email: ____________________

Date of Birth: __________ Soc. Sec. __________ Circle One: Male Female

Name of Parent or Guardian: ______________________________

Name of High School: ______________________________

High School Address: ______________________________

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Grade in School that will be completed by June 2010: ____________________

Please ask one of your mathematics teachers to write a letter of recommendation for you on the enclosed form.

Teacher's Name: ______________________________

Please tell us about yourself by answering the following questions. If you need more space, please attach another sheet of paper.

a. Have you ever participated in a special program in mathematics or science before? If so, tell us briefly about that experience.

b. Tell us about any other mathematical experiences you have had. For example, have you ever done a special mathematics project, or entered a mathematics competition?

c. What other interests and hobbies do you have?

d. What do you hope to gain by coming to PROMYS?

The cost of room and board for the six weeks will be $2,255. Instructional fees are an additional $445. No one should be discouraged from applying to PROMYS because of financial considerations. Thanks to support from our sponsors we are able to offer financial assistance to those who need it. PROMYS is committed to the principle that no one shall be unable to attend the program because of financial need. Please tell us how much financial assistance you require in order to participate in PROMYS this summer.

Can you afford to participate in PROMYS without any financial aid? ____________________

If not, how much could you afford to pay? ____

If you wish to be considered for financial aid, please ask a parent or guardian to complete the enclosed financial aid form.

Do you wish to identify yourself as a member of a racial or ethnic group? Yes _____ No _____

If so, to which group do you belong? ____

Please tell us how you learned of the PROMYS program. ______________________________

Applicant's signature: ________________________________ Date: __________________

The Problems

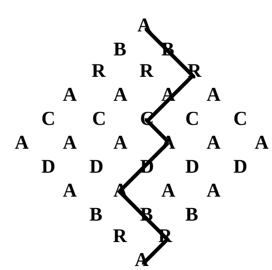
Please attempt each of the following problems. Though they can all be solved with no more than a standard high school mathematics background, most of the problems require considerably more ingenuity than is usually expected in high school. You should keep in mind that we do not expect you to find complete solutions to all of them. Rather, we are looking to see how you approach challenging problems. Here are a few suggestions:

- Think carefully about the meaning of each problem.
- Examine special cases, either through numerical examples or by drawing pictures.
- Be bold in making conjectures.
- Test your conjectures through further experimentation, and try to devise mathematical proofs to support the surviving ones.
- Can you solve special cases of a problem, or state and solve simpler but related problems?

If you think you know the answer to a question, but cannot prove that your answer is correct, tell us what kind of evidence you have found to support your belief. If you use books or articles in your explorations, be sure to cite your sources.

You may find that most of the problems require some patience. Do not rush through them. It is not unreasonable to spend a month or more thinking about the problems. It might be good strategy to devote most of your time to a small selection of problems which you find especially interesting. Be sure to tell us about progress you have made on problems not yet completely solved. **For each problem you solve, please justify your answer clearly and tell us how you arrived at your solution.**

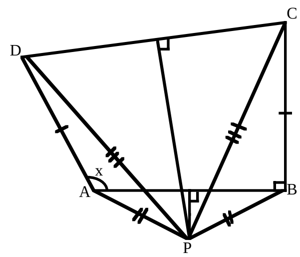
1. How many ways are there of spelling out "ABRACADABRA" by traversing the following diamond, always going from one letter to an adjacent one? One of the ways is shown.



2. In the game of Incan basketball, a points are given for a free throw and b points are given for a field goal, where a and b are positive integers. If $a = 2$ and $b = 5$, then it is not possible for a team to score exactly 1 point. Nor is it possible to score exactly 3 points. Are there any other unattainable scores? How many unattainable scores are there if $a = 3$ and $b = 5$? Is it true for any choice of a and b that there are only finitely many unattainable scores? Suppose a and b are unknown, but it is known that neither a nor b is equal to 2 and that there are exactly 65 unattainable scores. Can you determine a and b? Explain.

3. Show that there are no positive integers n for which $n^4 + 2n^3 + 2n^2 + 2n + 1$ is a perfect square. Are there any positive integers n for which $n^4 + n^3 + n^2 + n + 1$ is a perfect square? If so, find all such n.

4. According to the Journal of Irreproducible Results, any obtuse angle is a right angle! Here is their argument.



Given the obtuse angle x, we make a quadrilateral $ABCD$ with $\angle DAB = x$, and $\angle ABC = 90°$, and $AD = BC$. Say the perpendicular bisector to DC meets the perpendicular bisector to AB at P. Then $PA = PB$ and $PC = PD$. So the triangles PAD and PBC have equal sides and are congruent. Thus $\angle PAD = \angle PBC$. But PAB is isosceles, hence $\angle PAB = \angle PBA$. Subtracting, gives $x = \angle PAD - \angle PAB = \angle PBC - \angle PBA = 90°$. This is a preposterous conclusion – just where is the mistake in the "proof" and why does the argument break down there?

5. Consider a rectangular array of dots with an even number of rows and an even number of columns. Color the dots, each one red or blue, in such a way so that in each row half the dots are red and half are blue, and also in each column half are red and half are blue. Now, whenever two points of the same color are adjacent (in a row or column), join them by an edge of that color. Show that the number of red edges is the same as the number of blue edges.

6. The squares of an infinite chessboard are numbered as follows: in the first row and first column we put 0, and then in every other square we put the smallest non-negative integer that does not appear anywhere below it in the same column or anywhere to the left of it in the same row.

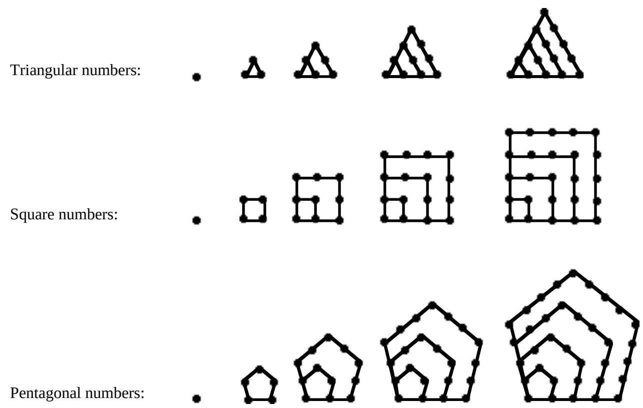
...	...						
6	7	...					
5	4	7	...				
4	5	6	7	...			
3	2	1	0	7	...		
2	3	0	1	6	7	...	
1	0	3	2	5	4	7	...
0	1	2	3	4	5	6	...

What number will appear in the 1000th row and 700th column? Can you generalize?

7. Let's agree to say that a positive integer is *prime-like* if it is not divisible by 2, 3, or 5. How many prime-like positive integers are there less than 100? less than 1000? A positive integer is *very prime-like* if it is not divisible by any prime less than 15. How many very prime-like positive integers are there less than 90000? Without giving an exact answer, can you say *approximately* how many very prime-like positive integers are less than 10^{10}? less than 10^{100}? Explain your reasoning as carefully as you can.

8. Find a positive integer m such that $\frac{1}{2}m$ is a perfect square and $\frac{1}{3}m$ is a perfect cube. Can you find a positive integer n for which $\frac{1}{2}n$ is a perfect square, $\frac{1}{3}n$ is a perfect cube and $\frac{1}{5}n$ is a perfect fifth power?

9. The triangular numbers are the numbers $1, 3, 6, 10, 15, \ldots$; the square numbers are the numbers $1, 4, 9, 16, 25, \ldots$ The pentagonal numbers $1, 5, 12, 22, 35, \ldots$. The geometrical language is justified by the following diagrams:



a. What are the first five hexagonal numbers? What are the first five septagonal numbers? What are the first five r-gonal numbers? Give a formula for the nth triangular number. Give a formula for the nth square number. Give a formula for the nth pentagonal number. In general, give a formula for the nth r-gonal number.

b. How many numbers can you find that are simultaneously triangular and square? How many numbers can you find that are simultaneously square and pentagonal?

10. The tail of a giant kangaroo is attached by a giant rubber band to a stake in the ground. A flea is sitting on top of the stake eyeing the kangaroo (hungrily). The kangaroo sees the flea leaps into the air and lands one mile from the stake (with its tail still attached to the stake by the rubber band). The flea does not give up the chase but leaps into the air and lands on the stretched rubber band one inch from the stake. The giant kangaroo, seeing this, again leaps into the air and lands another mile from the stake (i.e., a total of two miles from the stake). The flea is undaunted and leaps into the air again, landing on the rubber band one inch further along. Once again the giant kangaroo jumps another mile. The flea again leaps bravely into the air and lands another inch along the rubber band. If this continues indefinitely, will the flea ever catch the kangaroo? (Assume the earth is flat and continues indefinitely in all directions.)

11. Now that you have tried all of the problems, tell us which problem appealed to you the most and why.

PROMYS

Recommendation Form

To be completed by the applicant's mathematics teacher and mailed directly to PROMYS, Department of Mathematics, Boston University, 111 Cummington Street, Room 142, Boston, Massachusetts 02215; telephone: 617/353-2563.

Applicant's Name: ______________________________

Teacher's Name and School Address: ______________________________

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Telephone: ______________________________

Email: ______________________________

How long have you known the applicant? ______________________________

In what capacity? ______________________________

Please tell us how you learned of PROMYS. ______________________________

We would appreciate any comments you can make which might help us determine if the applicant would benefit from the PROMYS experience. Because of the highly intense nature of our program, we are especially interested in the applicant's motivation and potential for sustained hard work on challenging mathematical themes. Since PROMYS is a residential program in which many participants will be living away from home for the first time, we would also like to know your impressions of the applicant's emotional maturity. Please write your comments below (continued on back if necessary), and complete the table on the reverse side by checking the boxes you feel most appropriate. Thank you for your help.

(Over, Please)

Please complete the table below by checking the boxes you feel most appropriately describe the applicant.

	Top 1-2%	Top 5%	Top 10%	Top 25%	Top 50%	Not in top 50%
Interest in mathematics						
Ability to work independently						
Ability to work with others						
Imagination and creativity						
Analytical ability						
Personal initiative						
Perseverance						
Emotional maturity						

Teacher's signature: ______________________________ Date: ________________

PROMYS

Financial Aid Application Form

In order for us to consider the applicant for financial assistance, this form must be completed by a parent or guardian and mailed along with a copy of the 2009 federal income tax forms to: PROMYS, Department of Mathematics, Boston University, 111 Cummington Street, Room 142, Boston, Massachusetts 02215.

Applicant's name: ____________________

Parent/guardian name and address: ____________________

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Telephone: ____________________

Email: ____________________

What is your relationship to the applicant? ____________________

Adjusted gross income for 2009 (from IRS 1040 form): ____________________

Total untaxed income: ____________________

Please enter source(s) of untaxed income below.

____

____

How many people are in your family (including head(s) of household)? ____________________

In the space below, please explain any special circumstances you would like us to consider when reviewing the application for financial aid.

Signature: ____________________ Date: ____________________